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Assimilative Model for Ionospheric Dynamics Driven by TEC-related data from Beacon Satellites as well as by Skywave HF propagation Data from Multiple HF Channels

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The Ionospheric Reconstruction Problem: Tikhonov Method

$$N(\mathbf{r},t) = N_0(\mathbf{r},t)Q(u(\mathbf{r},t)); \quad \text{i.e. } Q(u(\mathbf{r},t)) = e^{u(\mathbf{r},t)}$$

$$U = \{u(\mathbf{r},t)\}$$

$$Y \approx M[U]$$

Y is the set of measured values obtained via various ionospheric measurements (such as TEC data, HF oblique propagation delay)

The solution must fit the data within errors of measurements. → $(Y - M[U])^T S^{-1}(Y - M[U]) / \dim(Y) \le 1$

There are infinitely many such solutions:

The smoothest solution is selected by minimizing the stabilizing functional → $U^T P^{-1} U \to \min$

-The pseudo-covariance *P* matrix is defined in such a way that the stabilizing functional tends to take on larger values for unreasonably behaving solutions (“reasonable” ⇔ “smooth”).

-The nonlinear optimization problem is solved iteratively (Newton-Kontorovich).

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HF Oblique Propagation Data within GPSII

-Simulated values of measured data can be obtained for any ionospheric model U via numerical ray tracing (RT).

-This defines the non-linear functional of measurements $M[U]$

Ray Tracing Equations

Hamiltonian Formulation of RT Equations [*Haselgrove*, 1957, *Jones*, 1975]

$$\frac{d\mathbf{R}}{d\tau} = -\frac{\partial H}{\partial \mathbf{k}} \Big/ \frac{\partial H}{\partial \omega} \; ; \qquad \frac{dg}{d\tau} = c$$

- Group path equation

$$\frac{d\mathbf{k}}{d\tau} = \frac{\partial H}{\partial \mathbf{R}} \Big/ \frac{\partial H}{\partial \omega} \; ; \qquad \frac{d\omega}{d\tau} = -\frac{\partial H}{\partial t} \Big/ \frac{\partial H}{\partial \omega}$$

- Doppler equation

$$\frac{d\mathbf{X}}{d\tau} = \mathbf{F}(\mathbf{X},[N, \frac{\partial N}{\partial t}]) \qquad \mathbf{X} = [X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8]^T$$

position: X_1, X_2, X_3

wave vector: X_4, X_5, X_6

group path: X_7

Doppler: X_8

HF Oblique Propagation Data within GPSII

-The non-linear inverse problem is solved iteratively as a sequence of linear problems. At the iteration *n* the non-linear functional $M[U]$ is approximated by a linear operator L as follows

$$M[U] = M[U_{n-1}] + L(U - U_{n-1}) + o\left(\left\|U - U_{n-1}\right\|\right) \qquad \Leftrightarrow L = \delta M / \delta U$$

- L is the Ray Path Response (RPR) operator

-The Ray Path Response operator L is estimated using extended RT equations – the equations augmented with the linearized ray-tracing equations

Extended RT Equations

$$\frac{d\mathbf{X}}{d\tau} = \mathbf{F}(\mathbf{X}, [N, \frac{\partial N}{\partial t}])$$

$$\frac{d\mathbf{A}}{d\tau} = \mathbf{B}(\mathbf{X}, [N, \frac{\partial N}{\partial t}])\mathbf{A} \qquad B_{ij} = \partial F_i / \partial X_j \Big|_{\mathbf{X}=\mathbf{X}(\tau)}$$

$$A_{ij}\Big|_{\tau=0} = \delta_{ij}, \quad i, j \in [1:8]$$

8+8x8=72 equations in the extended system

Evaluation of the Ray Path Response Operator *L*

The 3-D ionospheric model is specified by U_{n-1}

Perform ray-tracing (ray-homing) for the Rx-Tx configuration. Determine ray exit direction

Integrate the extended RT equations at the ray exit direction found at ray homing

Produce impulse response function $\boldsymbol{G}^{I}(\tau,\tau')$ of the linearized RT equations

Produce the Green function $\boldsymbol{G}(\tau,\tau')$ for the boundary-value problem (subject to the constraint that the ray end points are fixed in space)

Evaluate linear response $\mathbf{K}(\boldsymbol{\tau}')$ of parameters measured by the RX

Evaluate RPR

$$L^S = \int d\tau' \mathbf{K}(\tau') \left(\frac{\delta \mathbf{F}(\mathbf{X},[N,\dot{N}])}{\delta N} N_0 Q'(u_{n-1}) + \frac{\delta \mathbf{F}(\mathbf{X},[N,\dot{N}])}{\delta \dot{N}} \left(N_0 Q''(u_{n-1})\dot{u}_{n-1} + \dot{N}_0 Q'(u_{n-1}) \right) \right)$$

$$L^D = \int d\tau' \mathbf{K}(\tau') \frac{\delta \mathbf{F}(\mathbf{X},[N,\dot{N}])}{\delta \dot{N}} N_0 Q'(u_{n-1})$$

Where $N = N_0(\mathbf{r},t) Q(u_{n-1}(\mathbf{r},t))$

$$L\begin{bmatrix} \delta U \\ \delta \dot{U} \end{bmatrix} = L^S \,\delta U\big|_t + L^D \,\delta \dot{U}\big|_t$$

Assimilation of HF Data using RPR

Evaluation of RPR within the 3-D ionospheric inversion problem can be performed very efficiently as it is reduced to computation of several linear integrals

For HF data the main computational burden remains associated with the classical ray homing task. This task precedes computation of RPR

Numerical representation of the RPR is a sparse matrix with non-zero elements occupying only nodes of the spatial grid that are adjacent to the ray trajectory that connects receiver and transmitter. Matrix operations with RPR are not a substantial computational burden as we take advantage of the sparse character of RPR.

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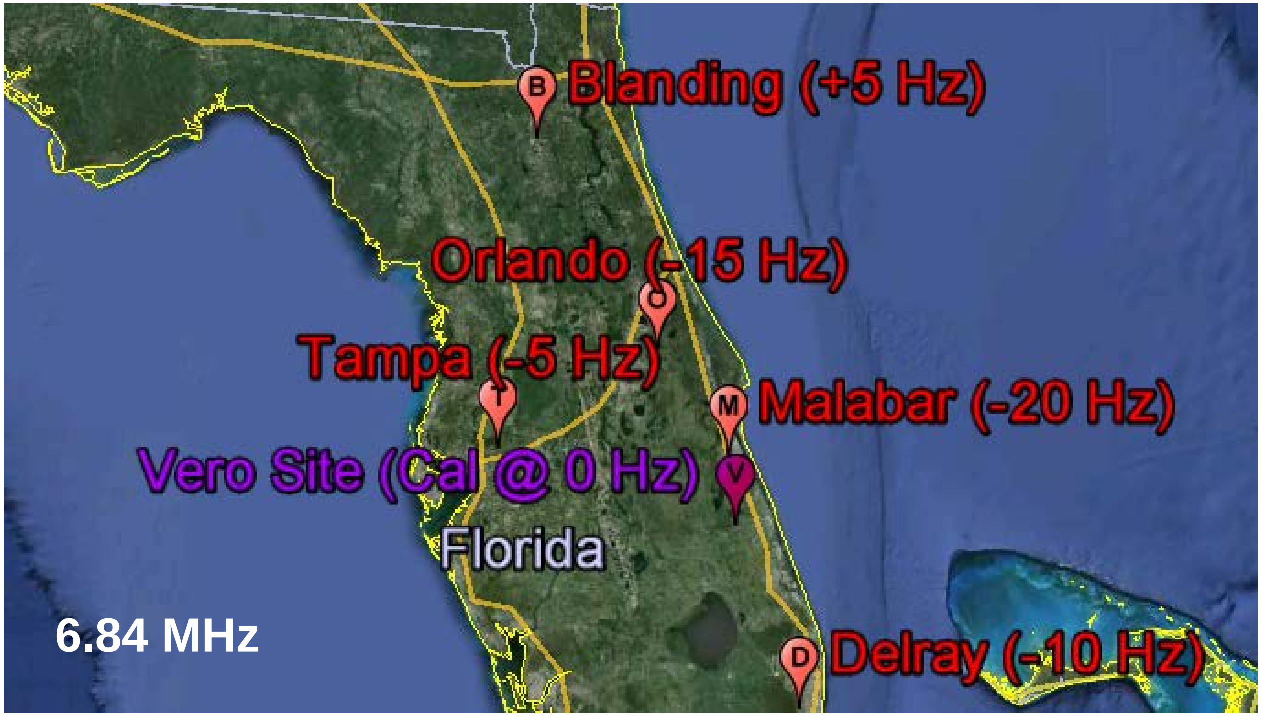
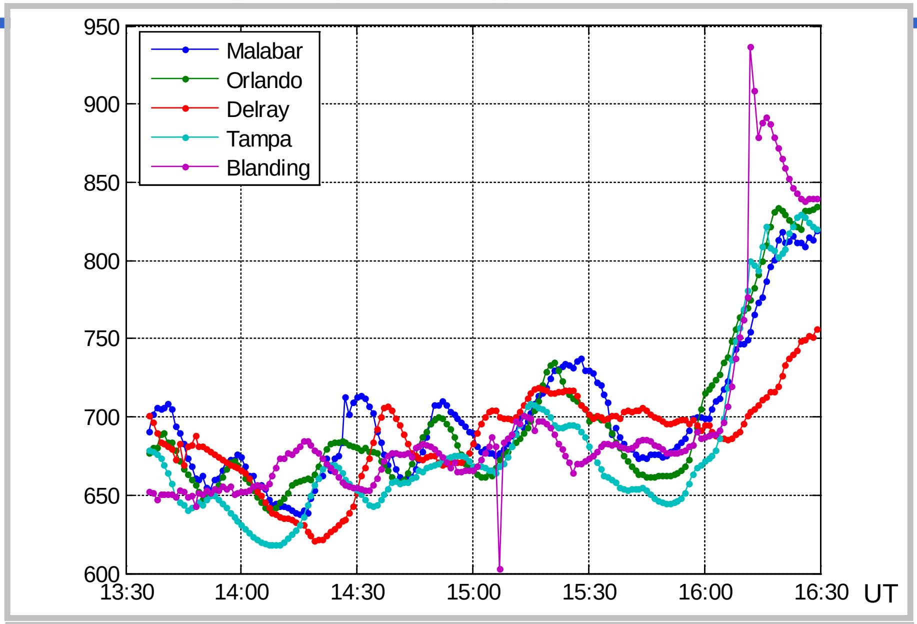
Range-Doppler Data Set



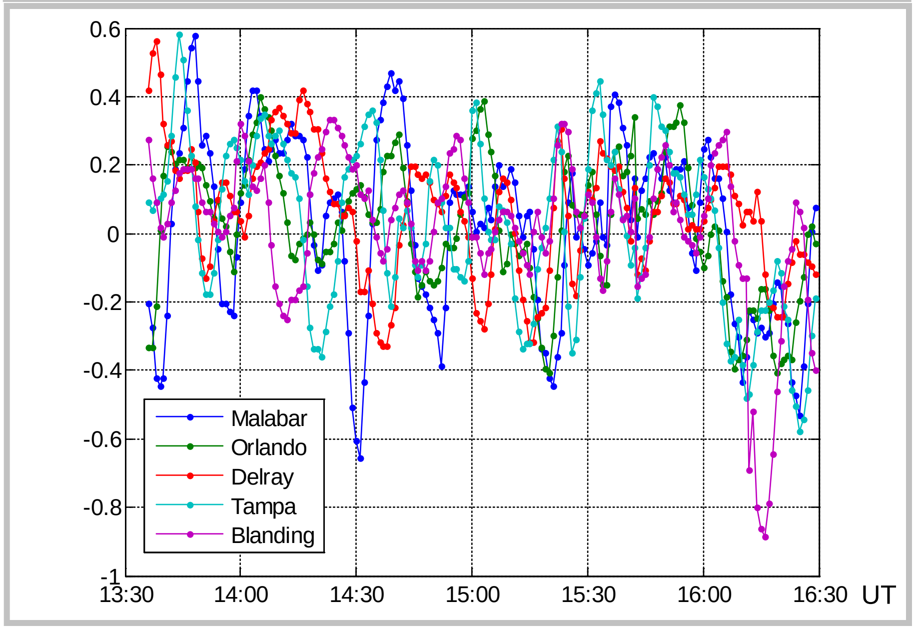
- **Florida Collection (August 13, 2013) Range/Doppler Data**
  - **Receiver at Vero Beach, FL**
  - **Multiple transmitter sites**
  - **Three hour collection of 3 KHz bandwidth FMCW waveform**

Range-Doppler Data of August 13, 2013 Employed by GPSII

Range data (km)



Doppler data (Hz)



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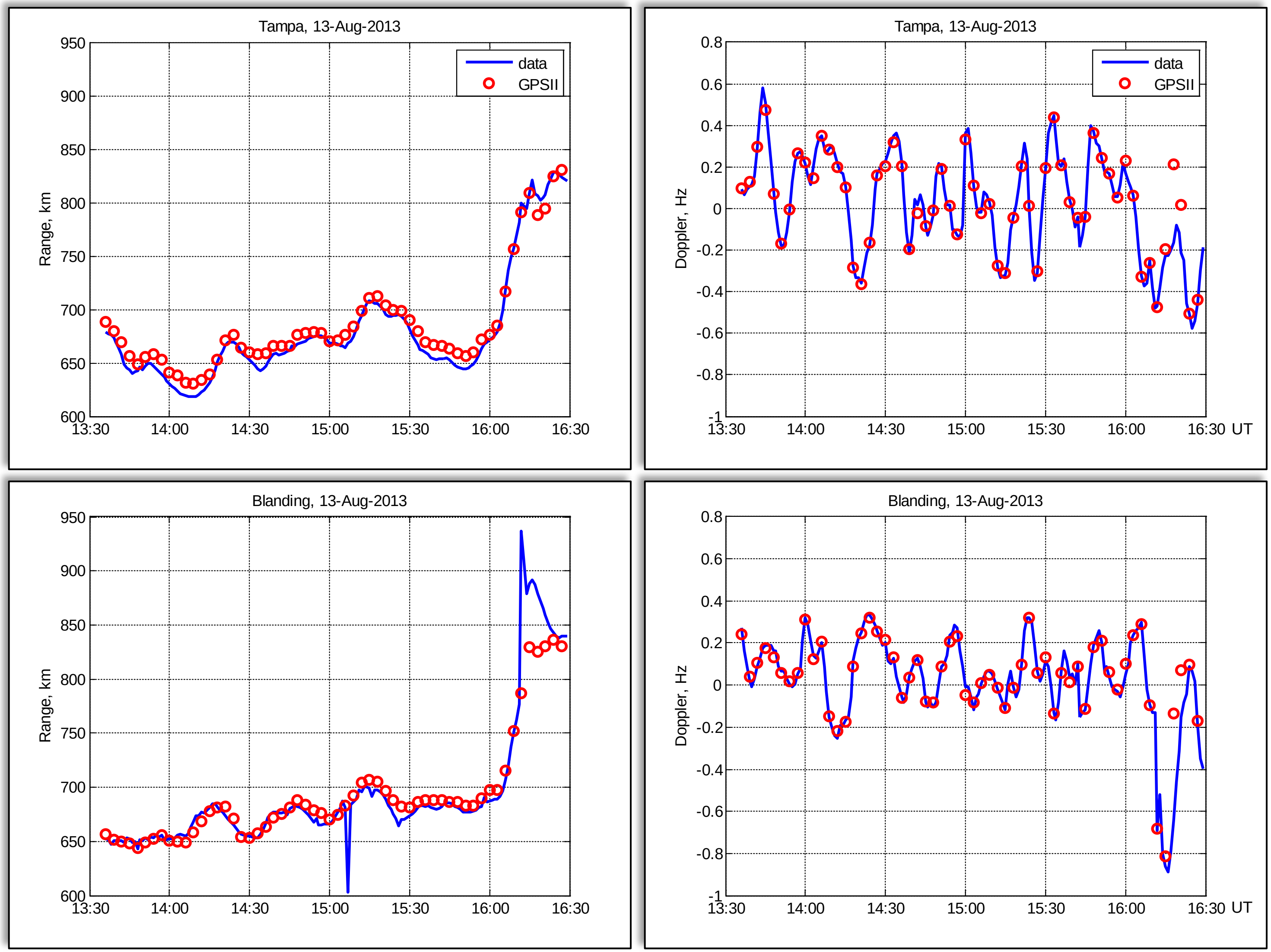
Range-Doppler Data Compared to GPSII Fit



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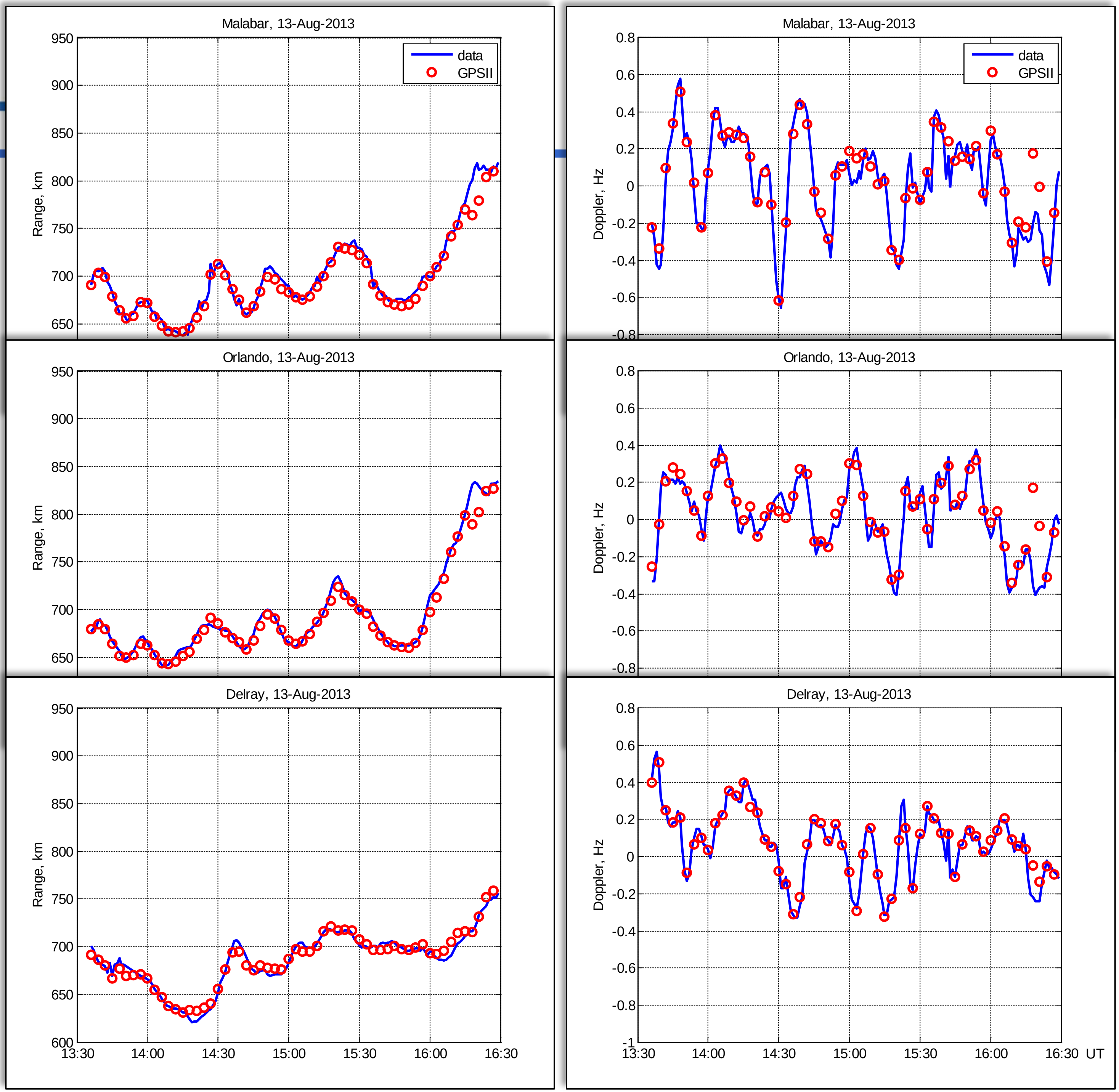
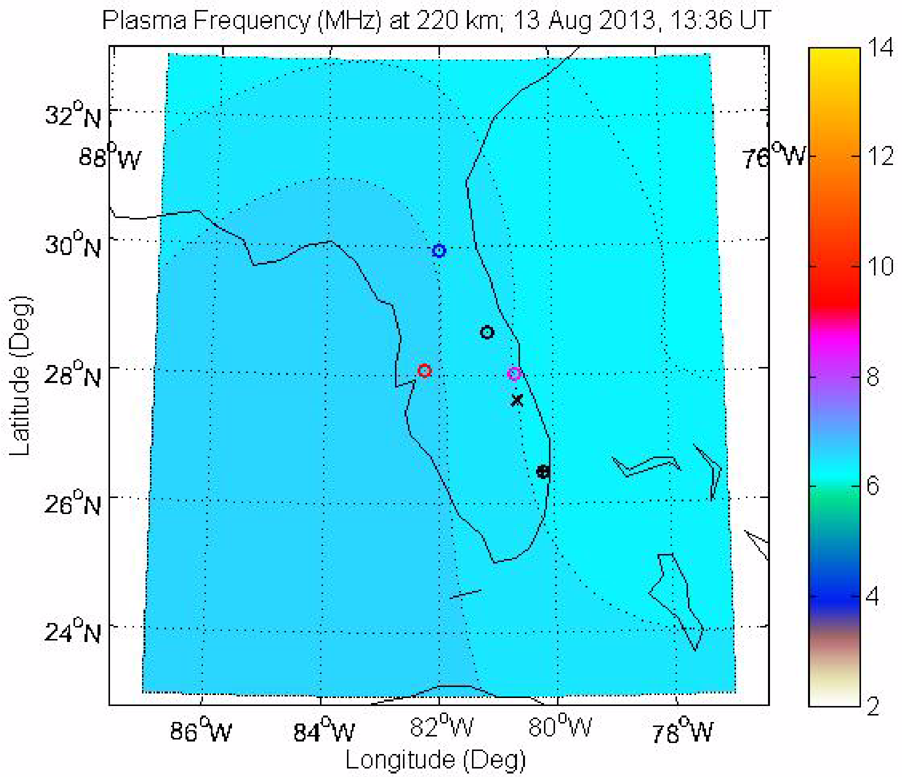
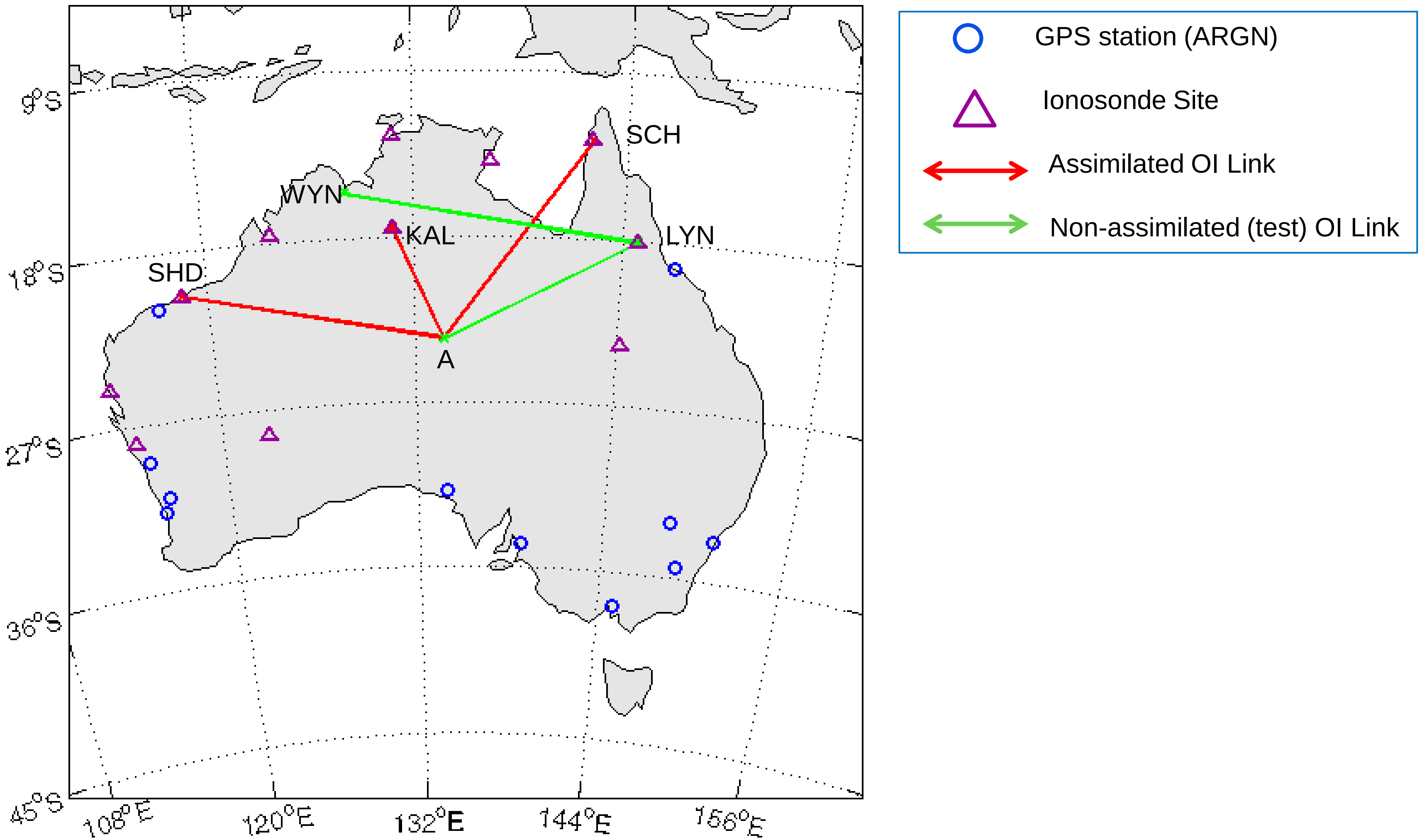


GPSII Solution with Range-Doppler Data



Assimilation of Oblique Ionograms (along with TEC and VI data)

Geography of DSTO data sources employed in this test



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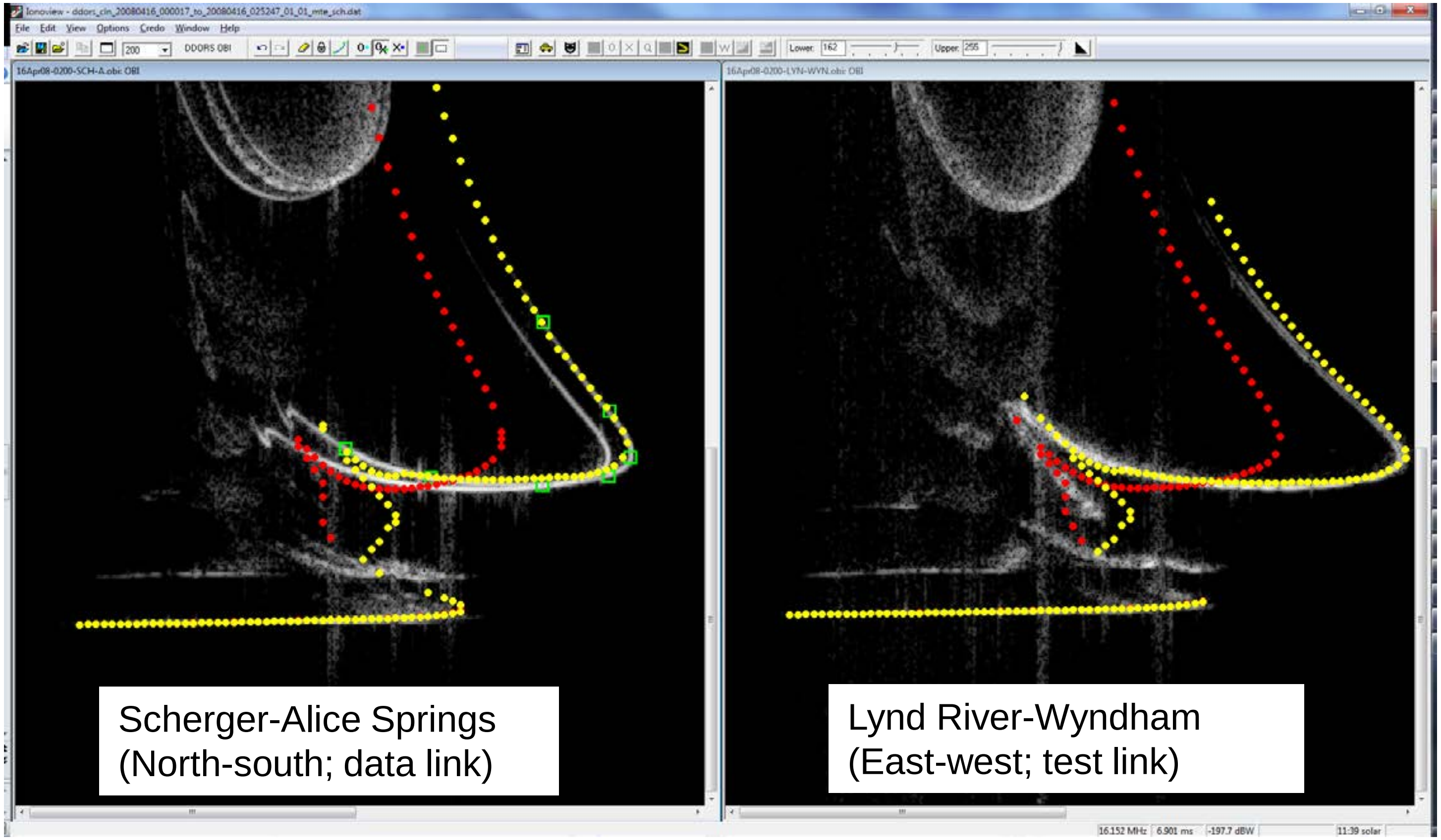
Comparison of GPSII results for 2 links

GPSII runs with oblique ionogram reproduce observed OI details (yellow) for both assimilated (left) and test (right) propagation links

Both links shown at 01:45. All synthetic OIs are extraordinary-ray traces.
(red dots represent climatological model prediction)



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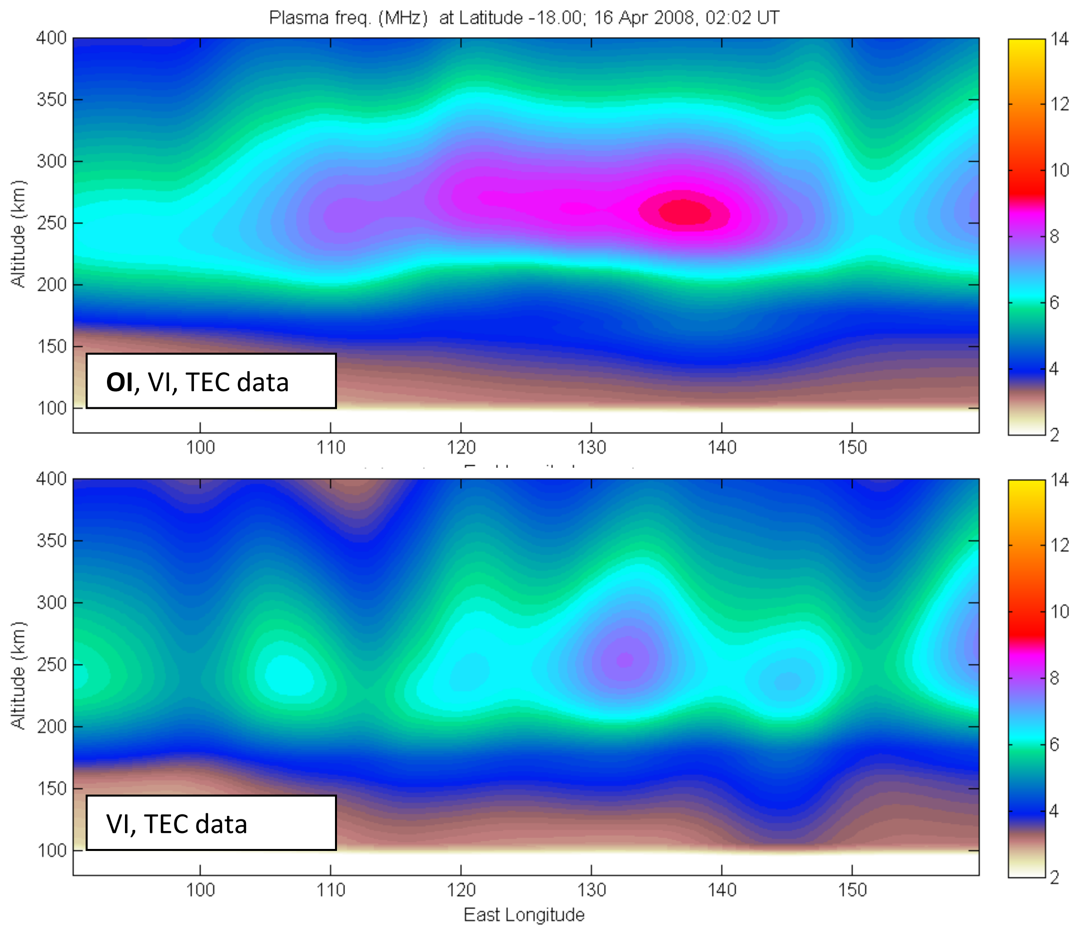
Impact of OI data on GPSII Model

Vertical cut through the model at latitude -18 degrees

Inversion with OI Data



Modification of GPSII to improve uniformity of fitting various data types

When GPSII operates with diverse data types it uses weighed average mean-square error as a measure of goodness of fit

$$(Y - MU)^T S^{-1}(Y - MU)/\dim(Y) \le 1$$

Y is the vector of measurements, MU is GPSII fit to the measurements, and S is the covariance matrix of measurement errors

Non-uniformity of the fit

When GPSII simultaneously matches TEC data and HF measurements it tends to over-fit TEC- and under-fit HF-measurements. Overall mean-square error of the fit remains of satisfactory value.

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In order to alleviate this undesirable phenomenon of non-uniform data fit we partition the vector of measurements into sub-vectors representing distinct categories of measured data

$$Y = \begin{bmatrix} Y^1 \\ Y^2 \\ . \\ . \\ Y^{N_c} \end{bmatrix}$$

$$\dim(Y) = \sum_{c=1}^{N_c} \dim(Y_c)$$

- **Introduce data-matching constraint for each category of measurements**

$$\frac{(Y^c - M^c U)^T S_c^{-1} (Y^c - M^c U)}{\dim(Y^c)} \le 1, \quad c = 1,2,.., N_c$$

The regularized problem:

$$U^T P^{-1} U + \sum_{c=1}^{N_c} \alpha_c \frac{(Y^c - M^c U)^T S_c^{-1} (Y^c - M^c U)}{\dim(Y^c)} \to \min$$

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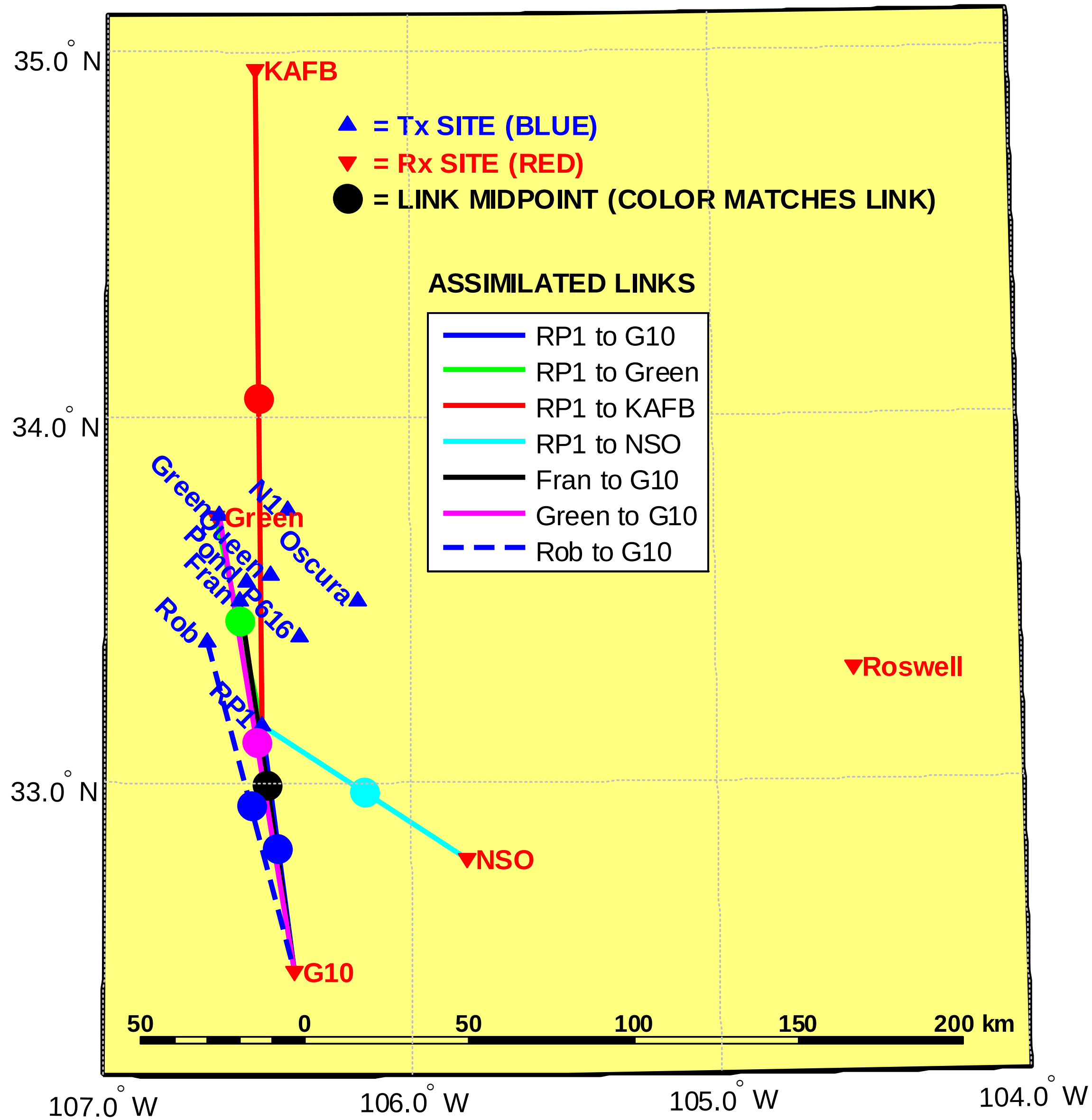
GPSII Data Categories

- Absolute TEC
- Relative TEC
- Profile of electron density
- Critical frequency of an electron density profile
- Critical height of an electron density profile
- HF propagation delay
- HF steer/bearing
- HF elevation
- HF Doppler
- HF maximum observable frequency (MOF)
- HF delay at MOF
- OTHR backscatter ionospheric Doppler
- Leading edge of backscatter ionogram
- Trailing edge of backscatter ionogram
- Other

Assimilated Delay-Doppler

- **HF Links used in the following results are shown here**



Evaluation of GPSII improvement using uniform fitting of various data types

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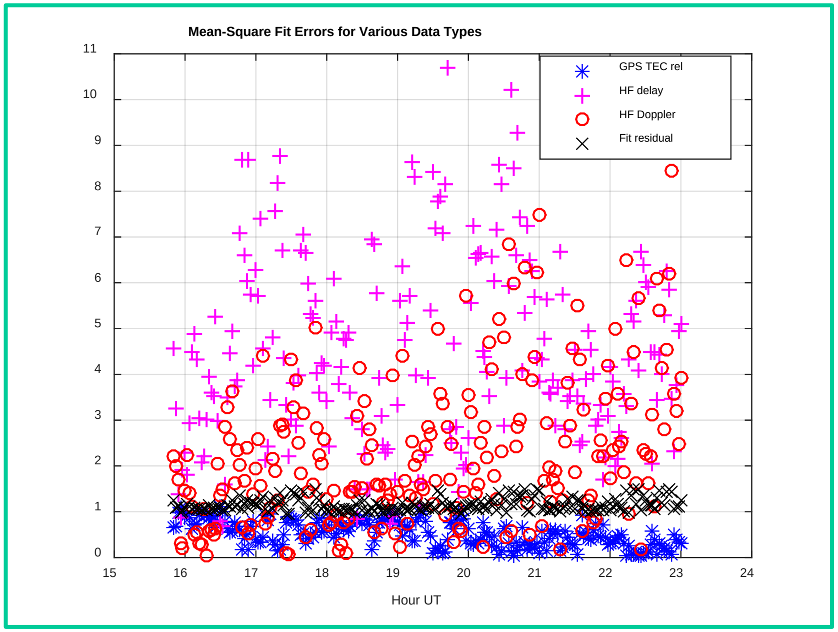
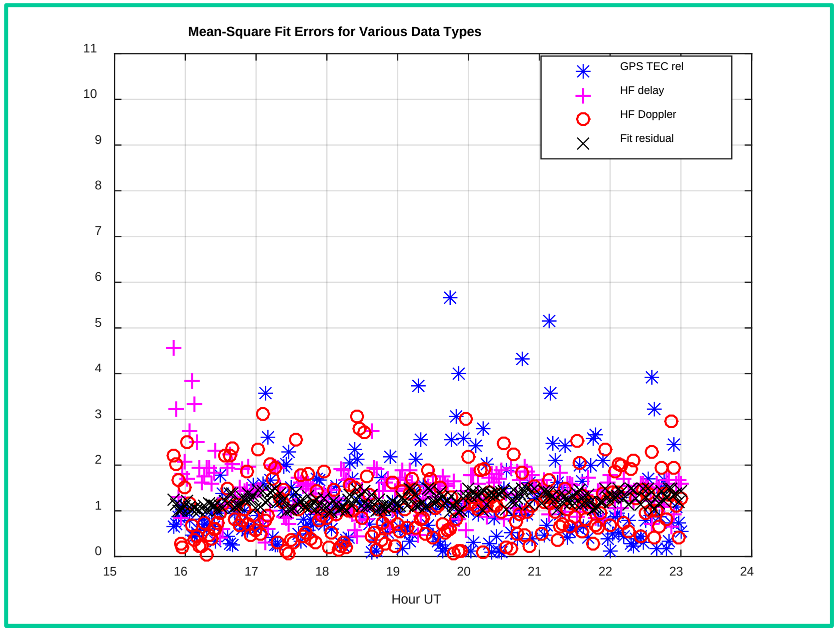
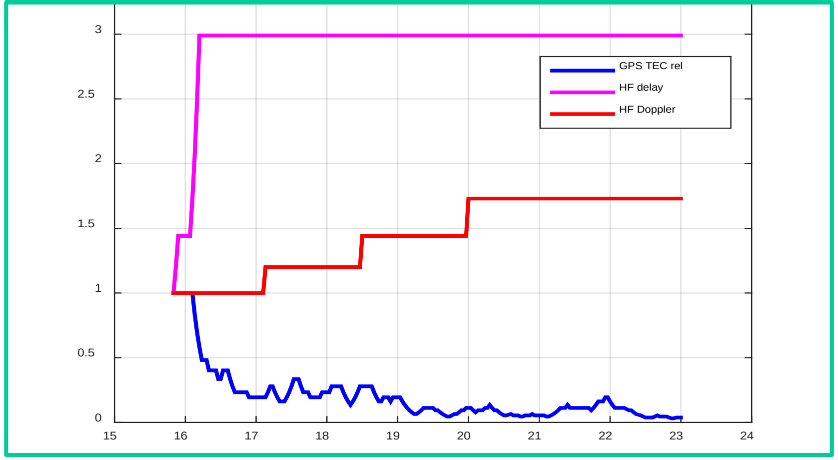
Legacy code driven by range, Doppler and TEC data



New code (uniform fit) driven by range, Doppler and TEC data



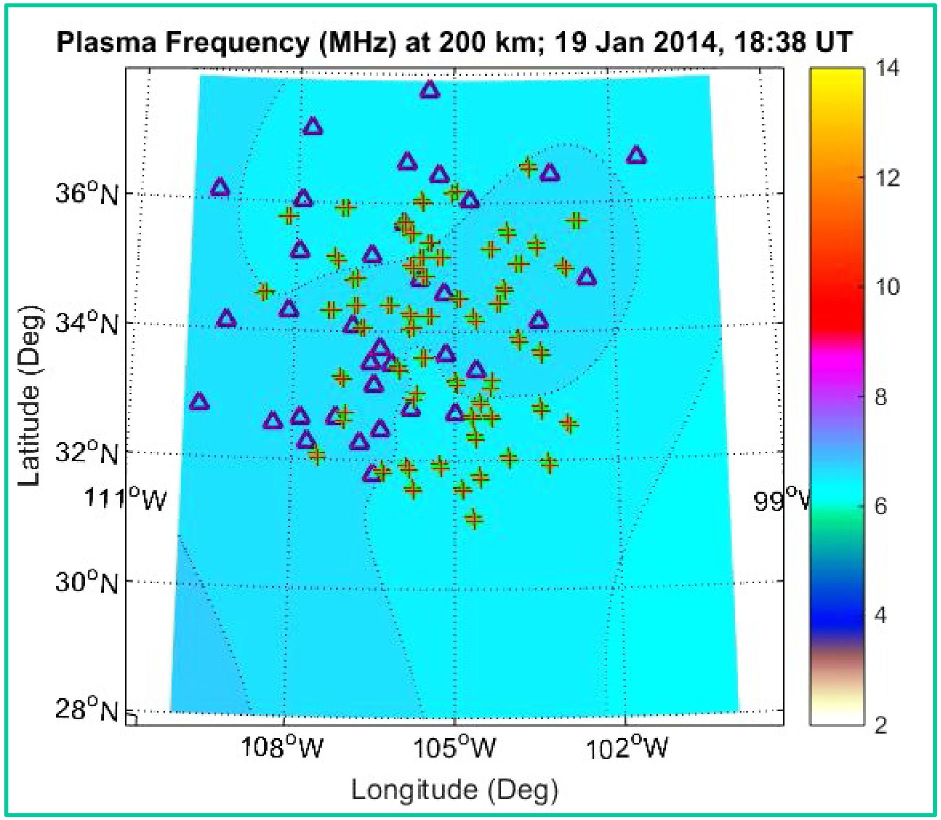


Mean-square fit errors for various data types

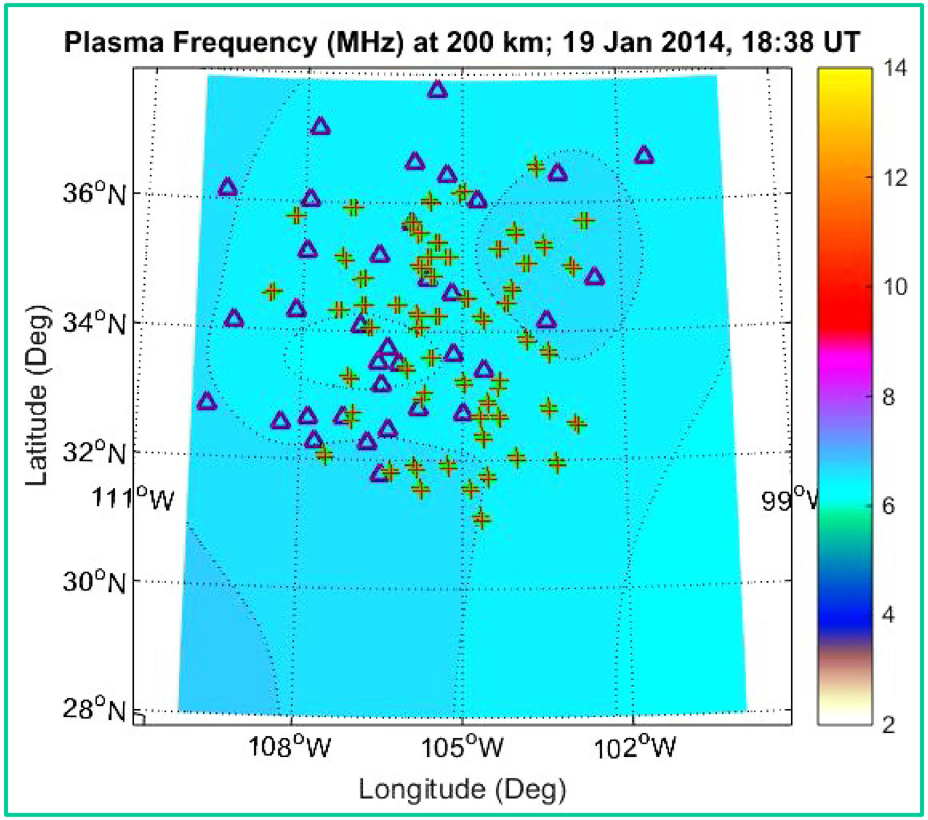
Weight Factors

Non-Uniform vs Uniform Fit of HF/GPS Data

GPSII maps of plasma frequency which also show positions of GPS receivers (triangles) and ionospheric pierce points of GPS lines of sight (plusses)



Solution with legacy algorithm



Solution with uniform fitting of diverse data

Uniform fitting of diverse data typically reveals ionospheric structure to greater detail

Evaluation of GPSII improvement using uniform fitting of various data types

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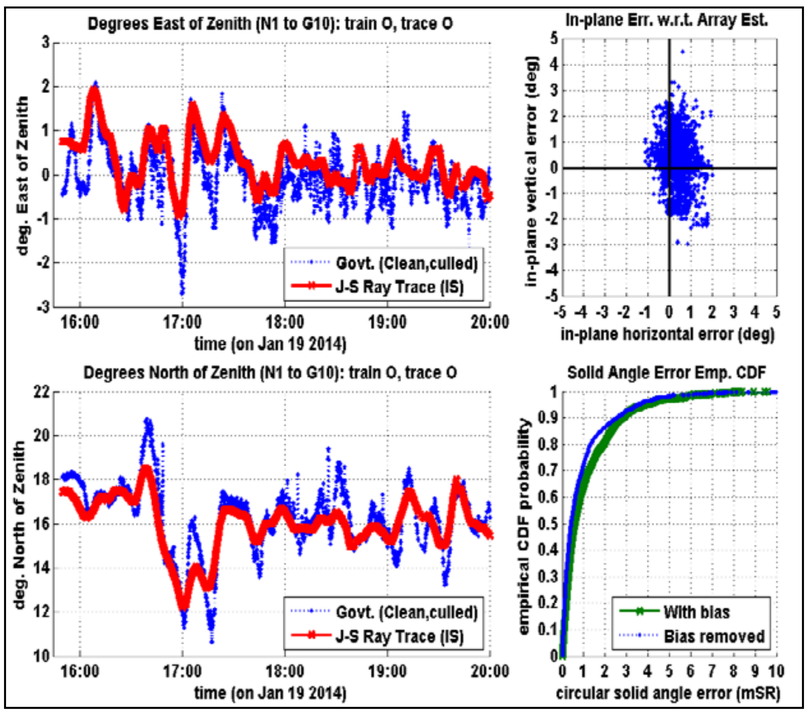
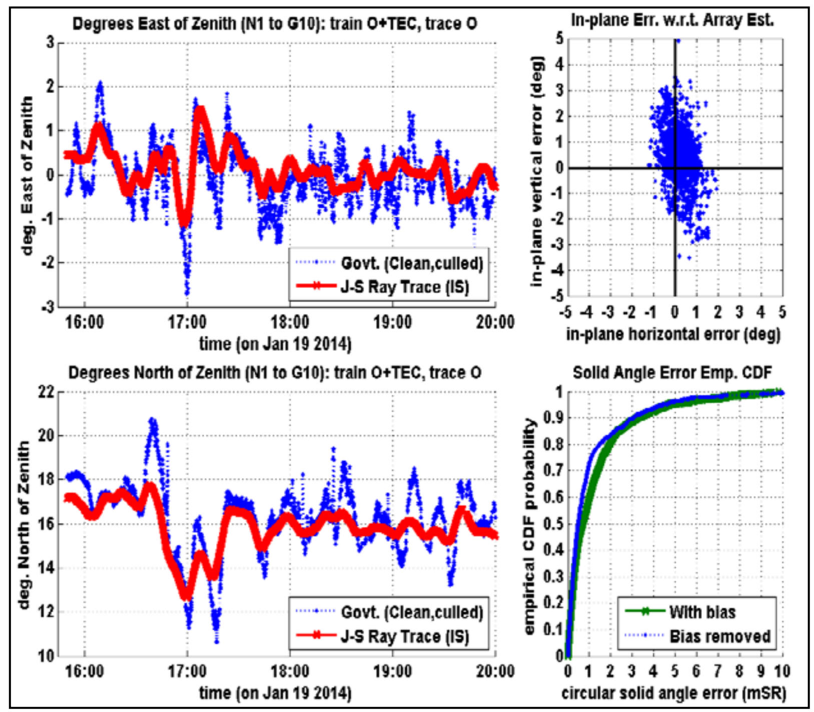
Legacy code driven by range and Doppler data



Legacy code driven by range, Doppler and TEC data



Evaluation of GPSII improvement using uniform fitting of various data types

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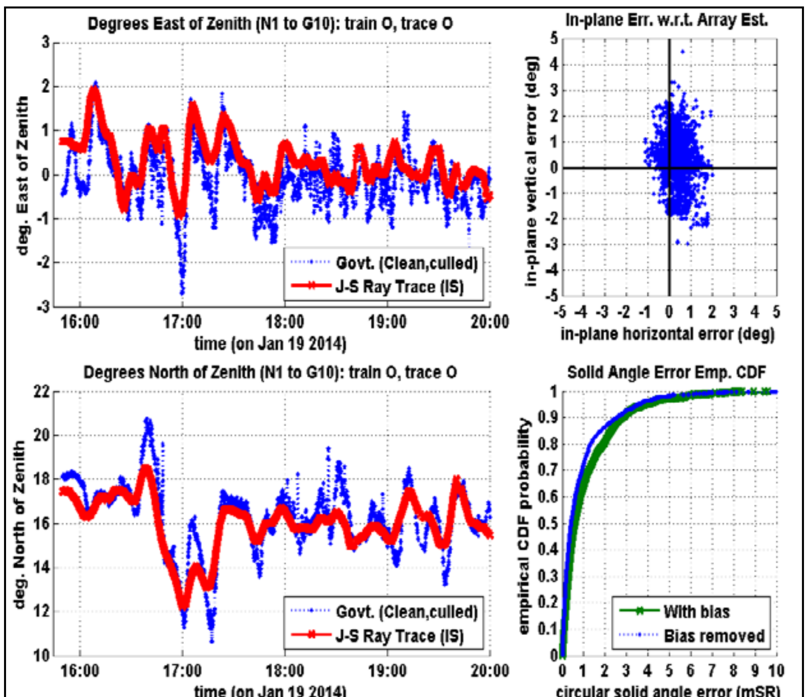
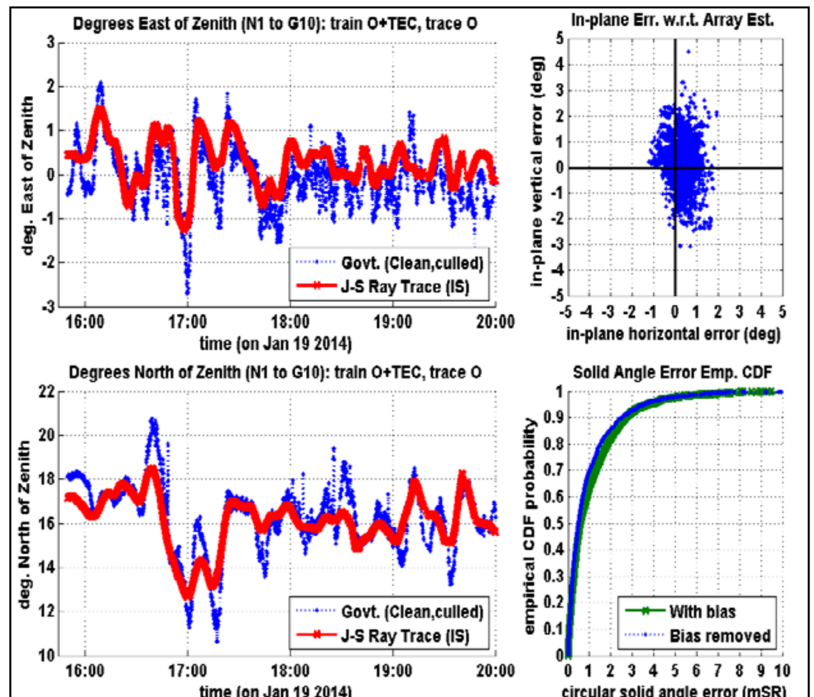
Legacy code driven by range and Doppler data



Code with weights rebalancing driven by range, Doppler, and TEC data



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Conclusions

- The theoretical framework for incorporating HF channel probe data (propagation delay, angles of arrival, Doppler shift) into ionospheric inversion algorithms has been developed
- Algorithm for achieving uniform fitting of diverse kinds of data within ionospheric inversion have been developed
- Capabilities for balanced assimilation of data from multiple HF channel probes, and GPS TEC receivers have been added to GPSII